6.1 Chapter 30, Problem 40.

A coaxial cable consists of a solid inner conductor of radius $a$ and a hollow outer conductor of inner radius $b$ and thickness $c$. The two carry equal but opposite currents $I$, uniformly distributed. Find expressions for the magnetic field strength as a function of radial position $r$ (a) within the inner conductor, (b) between the inner and outer conductors, and (c) beyond the outer conductor.

Solution
For a long, straight cable, the magnetic field can be found from Ampere’s law. The field lines are cylindrically symmetric and form closed loops, hence must be concentric circles, which we also choose as amperian loops. Take positive circulation counterclockwise so that positive current is out of the page. Then \[ \oint_c \mathbf{B} \cdot d\mathbf{l} = 2\pi r \mathbf{B} = \mu_0 I_{\text{encircled}}. \] Assume that the current density in each conductor is uniform; i.e., the current is proportional to the cross-sectional area. We may calculate $I_{\text{encircled}}$ in four regions of space. (a) For $r \leq R_a$, $I_{\text{encircled}} = I\left(\frac{r^2}{R_a^2}\right) = \frac{Ir}{R_a^2}$; so $B = \frac{\mu_0 I}{2\pi r}$. (b) For $R_a \leq r \leq R_b$, $I_{\text{encircled}} = I$, so $B = \frac{\mu_0 I}{2\pi r}$. (Although not asked for, for $R_b \leq r \leq R_c$, $I_{\text{encircled}} = 0$; so $B = 0$.

6.2 Chapter 30, Problem 51.

When a sample of a certain substance is placed in a 250.0-mT magnetic field, the field inside the sample is 249.6 mT. Find the magnetic susceptibility of the substance. Is it ferromagnetic, paramagnetic, or diamagnetic?

Solution
Equation 30-13 gives the relation between the internal and applied magnetic fields in terms of the relative permeability or the magnetic susceptibility. For the sample described in this problem, the latter is $\chi = (B_{\text{int}} - B_{\text{app}})/B_{\text{app}} = (249.6 - 250.0)/250.0 = -1.6 \times 10^{-3}$. Since $\chi < 0$ (or $B_{\text{int}} < B_{\text{app}}$) the material is diamagnetic.

6.3 Chapter 31, Problem 10.

A 1.8-m high runner sprints eastward at 9.5 m/s along the equator, where Earth’s magnetic field points horizontally with a strength of 31 \( \mu \)T. (a) What is the magnitude of the emf induced between the runner’s head and feet? (b) Which end is positive?

Solution
(a) The motional emf (on possible charge carriers in the runner’s body) is $(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{\ell} = \mathbf{v} \ell \mathbf{B}$, for $\mathbf{v}$, $\ell$, and $\mathbf{B}$ mutually perpendicular. (Of course, an actual sprinter would lean forward.) Thus, $|\mathcal{E}| = (9.5 \text{ m/s})(1.8 \text{ m}) \times (31 \mu \text{T}) = 0.530 \text{ mV}$. (b) With $\mathbf{v}$ east along the equator (\( \hat{i} \)) and $\mathbf{B}$ horizontally north (\( \hat{j} \)), $\mathbf{v} \times \mathbf{B}$ is vertically upward (\( \hat{k} \)), so the head of the runner is positive relative to his/her feet.

6.4 Chapter 31, Problem 25.

A credit-card reader extracts information from the card’s magnetic stripe as it is pulled past the reader’s head. At some instant the card motion results in a magnetic field at the head that is changing at the rate of 450 \( \mu \text{T/\text{ms}} \). If this field passes perpendicularly through a 5000-turn head coil 2.0 mm in diameter, what will be the induced emf?
Solution
The magnetic flux through the coil in the reader’s head is changing at a rate of \( NA dB/dt = (5000)\pi (1 \text{ mm})^2 (450 \mu \text{T}/\text{ms}) \), 7.07 mV. According to Faraday’s law, this is equal to the magnitude of the induced emf.

6.5 Chapter 31, Problem 27.

Figure 31-49 shows a pair of parallel conducting rails a distance \( \ell \) apart in a uniform magnetic field \( B \). A resistance \( R \) is connected across the rails, and a conducting bar of negligible resistance is being pulled along the rails with velocity \( v \) to the right. (a) What is the direction of the current in the resistor? (b) At what rate must work be done by the agent pulling the bar?

Solution
(a) The force on a (hypothetical) positive charge carrier in the bar, \( qv \times B \), is upward in Fig. 31-49, so current will circulate CCW around the loop containing the bar, the resistor, and the rails (i.e., downward in the resistor). (The force per unit positive charge is the motional emf in the bar.) Alternatively, since the area enclosed by the circuit, and the magnetic flux through it, are increasing, Lenz’s law requires that the induced current oppose this with an upward induced magnetic field. Thus, from the right-hand rule, the induced current must circulate CCW. (Take the positive sense of circulation around the circuit CW, so that the normal to the area is in the direction of \( B \), into the page.) (b) In Example 31-4, which analyzed the same situation, the current in the bar was found to be \( I = |E|/R = B\ell v/R \). Since this is perpendicular to the magnetic field, the magnetic force on the bar is \( F_{\text{mag}} = I\ell B \) (to the left in Fig. 31-49). The agent pulling the bar at constant velocity must exert an equal force in the direction of \( v \), and therefore does work at the rate \( F \cdot v = I\ell B v = (B\ell v)^2/R \). (Note: The conservation of energy requires that this equal the rate energy is dissipated in the resistor (we neglected the resistance of the bar and the rails), \( I^2 R = (B\ell v/R)^2 R \).)

6.6 Chapter 31, Problem 49.

Figure 31-58 shows an unusual design for a generator, consisting of a conducting bar that rotates about a central axis while making contact with a conducting ring of radius \( R \). A uniform magnetic field is perpendicular to the ring. Wires from the axis and ring carry power to a load. Find an expression for the emf induced in this generator when the bar rotates with angular speed \( \omega \).

Solution
Each rotation, the bar sweeps through an area of \( \pi R^2 \) perpendicular to the magnetic field \( B \), so the flux changes by \( \Delta \phi_B = \pi R^2 B \) in one period of rotation \( \Delta t = 2\pi/\omega \). Then the magnitude of the induced emf is \( E = \mid - \Delta \phi_B / \Delta t \mid = \pi R^2 B / (2\pi/\omega) = \frac{1}{2} \omega R^2 B \). Alternatively, at a point \( r \) on the bar, there is a motional emf resulting from an equivalent electric field of \( v \times B = -v B \hat{r} = -\omega r B \hat{r} \) (minus \( \hat{r} \) is toward the center). The emf developed across the length of the bar is \( E = \int_0^R E \cdot dr = -\omega B \int_0^R r \ dr = -\frac{1}{2} \omega B R^2 \) (the axis is positive relative to the rim).

6.7 Chapter 32, Question 1.

Figure 32-18 shows two pairs of identical coils in different geometrical arrangements. For which arrangement is the mutual inductance greatest? Why?

Solution (a) because the flux is maximized in both coils
6.8 Chapter 32, Question 7.

In a popular demonstration of induced emf, a light bulb is connected across a large inductor in an LR circuit, as shown in Fig 32-19. When the switch is opened, the bulb flashes brightly and burns out. Why?

Solution The inductors react to the change in current when the switch is opened, creating a large back current, which flows through the light.

6.9 Chapter 32, Question 14.

A 1-H inductor carries 10 A, and a 10-H inductor carries 1A. Which inductor contains more stored energy?

Solution Because energy is \( \frac{1}{2} LI^2 \), the 1 H inductor stores more energy.

6.10 Chapter 32, Question 15.

Does the energy density in a magnetic field depend on the direction of the field?

Solution Since the energy density is proportional to \( B^2 \), there is no dependence on the direction of \( B \).

6.11 Chapter 32, Problem 48.

Show that the quantity \( B^2/2\mu_0 \) has the units of energy density \( (J/m^3) \).

Solution The units of \( B^2/2\mu_0 \) are \( T^2 = (N/A^2 m^2) = J/m^3 \).

6.12 Chapter 33, Problem 1.

Much of Europe uses AC power at 230 V rms and 50 Hz. Express this AC voltage in the form of Equation 33-3, taking \( \phi = 0 \).

Solution Use of Equations 33-1 and 2 allows us to write \( V_p = \sqrt{2} V_{rms} = \sqrt{2}(230 \text{ V}) = 325 \text{ V}, \) and \( \omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ s}^{-1} \). Then the voltage expressed in the form of Equation 33-3 is \( V(t) = (325 \text{ V}) \times \sin([314 \text{ s}^{-1}]t) \).

6.13 Chapter 33, Problem 13.

What is the rms current in a 1.0-\( \mu \text{F} \) capacitor connected across the 120-V rms, 60-Hz AC line?

Solution Equation 33-5 can be used with the rms current and voltage, since both are \( 1/\sqrt{2} \) times their peak values. Thus, \( I_{rms} = \omega CV_{rms} = (2\pi \times 60 \text{ Hz})(1 \text{ \mu F})(120 \text{ V}) = 45.2 \text{ mA} \).

6.14 Chapter 33, Problem 53.

An electric drill draws 4.6 A rms at 120 V rms. If the current lags the voltage by 25°, what is the drill’s power consumption?

Solution The average power consumed by an AC circuit is given by Equation 33-17, \( P_{av} = V_{rms}I_{rms} \cos \phi = (120 \text{ V}) \times (4.6 \text{ A}) \cos(25°) = 500 \text{ W} \).

6.15 Chapter 33, Problem 58.

A rural power line carries 2.3 A rms at 4000 V. A stepdown transformer reduces this to 235 V rms to supply a house. Find (a) the turns ratio of the transformer and (b) the current in the 235-V line to the house.

Solution (a) The turns ratio given in Equation 33-18 is \( N_{sec}/N_{pri} = V_{sec}/V_{pri} = 235/4000 = 1/17 \). (b) If there are no transformer losses, Equation 33-19 gives \( I_{sec} = (V_{pri}/V_{sec})I_{pri} = (17)(2.3 \text{ A}) = 39.1 \text{ A (rms)} \).
6.16 Chapter 34, Problem 7.

At a particular point the instantaneous electric field of an electromagnetic wave points in the $+y$ direction, while the magnetic field points in the $-z$ direction. In what direction is the wave propagating?

**Solution** For electromagnetic waves in vacuum, the directions of the electric and magnetic fields, and of wave propagation, form a right-handed coordinate system, as shown. (The vector relationship is summarized in Equation 34-20b.) Therefore, the given wave is headed in the $-x$-direction.

6.17 Chapter 34, Problem 12.

A light-minute is the distance light travels in one minute. Show that the Sun is about 8 light-minutes from Earth.

**Solution** A light-minute (abbrev. c-min) is about $(3 \times 10^8 \text{ m/s})(60 \text{ s}) = 1.8 \times 10^{10} \text{ m}$, so the mean distance of the Earth from the Sun (an Astronomical Unit) is about $(1.5 \times 10^{11} \text{ m})/(1.8 \times 10^{10} \text{ m/c-min}) = 8.33 \text{ c-min}$. 
